Dielectrics - II

In the earlier lecture note we have studied that induced dipole moment $\vec{p}$ of the atom in presence of small electric field $\vec{E}$ is given by

$$\vec{p} = \alpha \vec{E} \quad \text{--- (i)}$$

where α is atomic polarizability. In above relation $\vec{p}$ and $\vec{E}$ points in the same direction. If electric field is applied at some angle to the axes, the dipole may be written in the following form

$$\vec{p} = \alpha_{\perp} \vec{E_{\perp}} + \alpha_{\parallel} \vec{E_{\parallel}} \quad \text{--- (ii)}$$

where we have resolved the $\vec{E}$ into parallel and perpendicular components and $\alpha_{\parallel}$ and $\alpha_{\perp}$ are the atomic polarizability corresponding to the parallel and perpendicular components.

For an asymmetrical molecule we can generalize the relation (ii) in the form given by

$$p_i = \sum_{j=x,y,z} \alpha_{ij} E_j \quad \text{--- (iii) a}$$

or

$$p_x = \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z$$

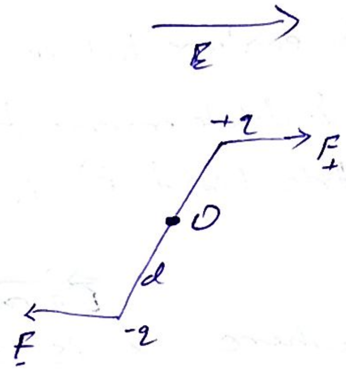
$$p_y = \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z \quad \text{--- (iii) b}$$

$$p_z = \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z$$

or in matrix form as

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad \text{--- (iii) c}$$

In equation [(iii) a], α_{ij} is called the polarizability tensor for the molecule.



Torque:

For the uniform electric field $\vec{E}$, torque $\vec{N}$ is given by

$$\vec{N} = (\vec{r}_+ \times \vec{F}_+) + (\vec{r}_- \times \vec{F}_-)$$

$$= \left[\frac{\vec{d}}{2} \times q\vec{E}\right] + \left[-\frac{\vec{d}}{2} \times (-q\vec{E})\right]$$

or $\vec{N} = q\vec{d} \times \vec{E}$

$$\boxed{\vec{N} = \vec{p} \times \vec{E}} \qquad \vec{p} = q\vec{d}$$

Polarization: We define dipole moment per unit volume as polarization denoted by $\vec{P}$.

Q. According to equation (i) the induced dipole moment of an atom is proportional to the external field. This is a "rule of thumb" not a fundamental law, and it is easy to concoct exceptions in theory. Suppose, for example the charge density of the electron cloud were proportional to the distance from the center, out of a radius R. To what power of E would p be proportional in that case? Find the condition $\rho(r)$ such that equation (i) will hold in the weak-field limit.

Hint: Consider $\rho(r) = \beta_0 r$, where $\beta_0 \rightarrow$ constant. apply Gauss's law to find E.